Resistive wall impedance and tune shift for a chamber with a finite thickness

Yoshihiro Shobuda and Kaoru Yokoya
KEK, High Energy Accelerator Research Organization, 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan
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Since the resistive wall impedance for a beam pipe of a nonround cross section depends on the coordinates of a witness particle, the witness particle receives an incoherent tune shift. When the expression for the impedance of an infinitely thick chamber is applied to the calculation of this tune shift, it becomes infinite. We have derived the resistive wall impedance for a chamber with a finite thickness and calculated the tune shift. There is no ambiguity in this expression for the tune shift, because it is automatically finite.

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I. INTRODUCTION

Resistive wall impedances have been studied in accelerator physics. Gluckstern, van Zeijts, and Zotter (GZZ) derived formulas of impedances for the case of elliptical and rectangular cross section when the position of the source particle and the witness particle are the same, and the thickness of the beam pipe is infinite [1]. Yokoya gave a computer algorithm to calculate the impedances for a general cross section when the thickness of the beam pipe is infinite [2].

It is known that a beam pipe of nonround cross section causes an incoherent tune shift, because the resistive wall impedance due to the source particle depends not only on the coordinates of the source particle, but also on those of the witness particle [1,2]. However, this tune shift becomes infinite when the impedances for a chamber with an infinite thickness are applied to its calculation, because the resistive wall wake function is proportional to $1/\sqrt{s}$ where s is the distance between the source particle and the witness particle. In previous studies, the tune shift was calculated by introducing an artificial cutoff [3,4]. This kind of cutoff causes an ambiguity in the calculation of the tune shift. It is possible to avoid this kind of infinity by considering the thickness of the beam pipe. The electromagnetic fields leak out when the thickness of the beam pipe is finite. Since the skin depth of the beam pipe material is proportional to $(1/\sqrt{k})$ where k is a wave number, this leak actually occurs on a large time scale. For this case, an important parameter is the thickness of the beam pipe instead of the skin depth.

It is important to consider the effect of the thickness of the beam pipe, and to understand how the divergence of the tune shift can be resolved. In Sec. II, we show how we include the effect of the thickness of the chamber to the resistive impedances. In Sec. III, we explicitly show the impedances and the tune shift. In Sec. IV, we apply our theory to KEKB electron ring and compare the experimental data with our theory. A summary is given in Sec. V.

II. DERIVATION OF IMPEDANCES FOR A CHAMBER WITH A FINITE THICKNESS

We assume throughout that the beam pipe is uniform longitudinally and the beam is ultrarelativistic. Since all of the field quantities which are effective on the wake field are proportional to $\exp[jk(ct-z)]$ where c is the velocity of light, we omit this factor to define the fields.

We denote by E_z the longitudinal component of the electric field generated by a source particle. Once the field on the inner surface of the pipe is known, E_z can be calculated by the Kirchhoff integral formula [2,5],

$$E_z(\mathbf{r}) = \oint [\boldsymbol{\nabla}'_\perp E_z(\mathbf{r}')G(\mathbf{r},\mathbf{r}') - E_z(\mathbf{r}')\boldsymbol{\nabla}'_\perp G(\mathbf{r},\mathbf{r}')]\cdot \mathbf{n}ds', \tag{1}$$

where $\oint$ is the integral along the boundary surface, the prime denotes quantities at the boundary, $\mathbf{n}$ is an outwardly directed normal to the boundary surface and $\boldsymbol{\nabla}_\perp$ is the two-dimensional gradient. The function G is the Green function satisfying

$$\Delta_\perp G(\mathbf{r},\mathbf{r}') = -\delta(\mathbf{r}-\mathbf{r}'). \tag{2}$$

Since the Kirchhoff integral is valid for any Green function satisfying Eq. (2), we choose the function with its value equal to zero at the boundary. By using this Green function, we can calculate the longitudinal impedance when we only know E_z at the boundary.

When E_z is obtained, the transverse force $F_\perp$ is calculated using the Panofsky-Wenzel theorem [6],

$$F_\perp(x,y) = -\frac{1}{jk}\boldsymbol{\nabla}_\perp E_z(x,y). \tag{3}$$

Thus, it is necessary to know E_z at the inner surface of the chamber for the case of a finite chamber thickness. The tangential (azimuthal) magnetic component (H_t) is calculated by

$$Z_0 H_t = E_n = -\boldsymbol{\nabla}_\perp \Phi(x,y), \tag{4}$$

$$\Delta_\perp \Phi(x,y) = -Z_0\delta(\mathbf{r}-\mathbf{r}_1), \tag{5}$$

where E_n is the normal component of the electric field, Z_0 is the impedance of free space and $\mathbf{r}_1$ is the transverse coordinate of the source particle. In order to obtain E_z at the boundary, we must find the relation between E_z and H_t along the boundary when the thickness of the chamber is finite.

We consider the case when the radius of the beam pipe is sufficiently larger than the skin depth. In order to find the relation between E_z and H_t at the boundary, it is sufficient to

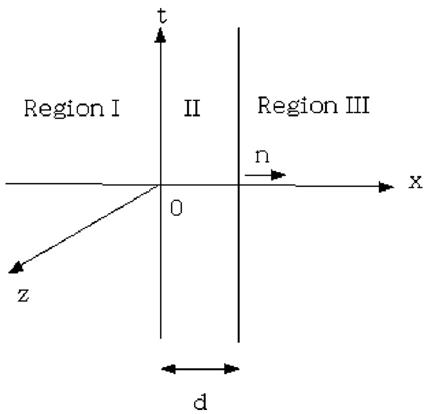

FIG. 1. A wall with its thickness d. The beam runs in region I.

consider the one-dimensional problem. Consider a wall with thickness d (see Fig. 1). We call $(x<0)$ region I, $(0<x<d)$ region II, and $(d<x<\infty)$ region III. We assume that the beam runs in region I. The beam creates fields on the inner surface of the wall $(x=0)$, which are written as $E_z(0)$ and $H_t(0)$. In region II, Maxwell equations are written as follows:

$$jkH_t(x)=(\sigma+jkc\epsilon_0)E_n(x),$$

$$\epsilon_0\frac{\partial E_n(x)}{\partial x}-jk\epsilon_0E_z(x)=0,$$

$$-\frac{\partial E_z(x)}{\partial x}-jkE_n(x)=-jkc\mu_0H_t(x),$$

$$\frac{\partial H_t(x)}{\partial x}=(\sigma+jkc\epsilon_0)E_z(x). \tag{6}$$

According to Eq. (6), $E_z(x)$ must satisfy

$$\frac{\partial^2E_z(x)}{\partial x^2}-\frac{2j}{\delta^2}E_z(x)=0, \tag{7}$$

where $\delta=\sqrt{2\rho_0/k}$ and $\rho_0=1/\mu_0c\sigma$ ($\rho_0\simeq10^{-10}$ m for a copper chamber at room temperature). The solutions for region II are

$$E_z(x)=c_1e^{\sqrt{2j}x/\delta}+c_2e^{-\sqrt{2j}x/\delta}, \tag{8}$$

$$H_t(x)=\frac{\sqrt{2j}}{\delta}\,\frac{1+jk\rho_0}{jkZ_0}(c_1e^{\sqrt{2j}x/\delta}-c_2e^{-\sqrt{2j}x/\delta}). \tag{9}$$

The boundary conditions are as follows:

$$E_z(0)=c_1+c_2, \tag{10}$$

$$0=c_1e^{\sqrt{2j}d/\delta}+c_2e^{-\sqrt{2j}d/\delta}, \tag{11}$$

$$H_t(0)=\frac{\sqrt{2j}}{\delta}\,\frac{1+jk\rho_0}{jkZ_0}(c_1-c_2), \tag{12}$$

which give the following relations:

$$E_z(0)\simeq-k\delta Z_0\frac{1+j}{2}\mathcal{G}H_t(0), \tag{13}$$

$$\mathcal{G}=\frac{e^{\sqrt{2j}d/\delta}-e^{-\sqrt{2j}d/\delta}}{e^{\sqrt{2j}d/\delta}+e^{-\sqrt{2j}d/\delta}}, \tag{14}$$

in normal applications where $k\rho_0\ll1$. For $d\to\infty$, Eq. (14) reproduces the relation

$$E_z(0)\simeq-k\delta Z_0\frac{1+j}{2}H_t(0), \tag{15}$$

which is already known [1]. We expect that Eq. (13), which is obtained in the one-dimensional case, is almost satisfied for the general beam-pipe case when the radius of the beam pipe is sufficiently larger than the skin depth [but, the thickness of the chamber (d) may be small].

In order to confirm that this expectation is plausible, we can consider a simple example for the two-dimension case where the cross section of the pipe is round (the radius of the pipe is b_0 and the thickness of the chamber is d). This problem was already solved by Chao [6]. The boundary conditions also give Eq. (13) when $b_0\gg\delta$ and $k\rho_0\ll1$. These situations suggest that Eq. (13) is almost satisfied for the general beam-pipe case, when the skin depth is sufficiently smaller than the radius of the beam pipe.

III. WAKE FUNCTIONS AND TUNE SHIFT

According to Eqs. (1) and (13), we find that E_z for a chamber with a finite thickness is that with an infinite thickness multiplied by the $\mathcal{G}$ factor, because H_t does not depend on the properties of wall materials. We obtain

$$E_z(x,y)=-k\delta(1+j)\frac{e^{\sqrt{2j}d/\delta}-e^{-\sqrt{2j}d/\delta}}{e^{\sqrt{2j}d/\delta}+e^{-\sqrt{2j}d/\delta}}\,\frac{Z_0}{2\pi b^3}\left(\frac{b^2}{2}D_0+D_{1x}x_1x+D_{1y}y_1y+D_{2xy}\frac{x^2-y^2}{2}+\cdots\right), \tag{16}$$

$$F_\perp(x,y)=\frac{\delta(1+j)}{j}\,\frac{e^{\sqrt{2j}d/\delta}-e^{-\sqrt{2j}d/\delta}}{e^{\sqrt{2j}d/\delta}+e^{-\sqrt{2j}d/\delta}}\,\frac{Z_0}{2\pi b^3}D_\perp,$$

$$D_\perp=\left\{\begin{pmatrix}D_{1x}x_1\\D_{1y}y_1\end{pmatrix}+D_{2xy}\begin{pmatrix}x\\-y\end{pmatrix}+\cdots\right\}, \tag{17}$$

where $2b$ is a typical vertical size of the cross section of the chamber; the analytic form of the D's is given in the Appendix for elliptic and rectangular cases. ($D_\perp$ is relevant to the tune shift. D_{1x}, D_{1y}, and D_{2xy} are equal to $\partial W_x/\partial x_s$, $\partial W_y/\partial y_s$, and $\partial W_x/\partial x_w$ in Fig. 8 of Ref. [2], respectively. We should notice that D_{2xy} vanishes when the cross section of the chamber is round.)

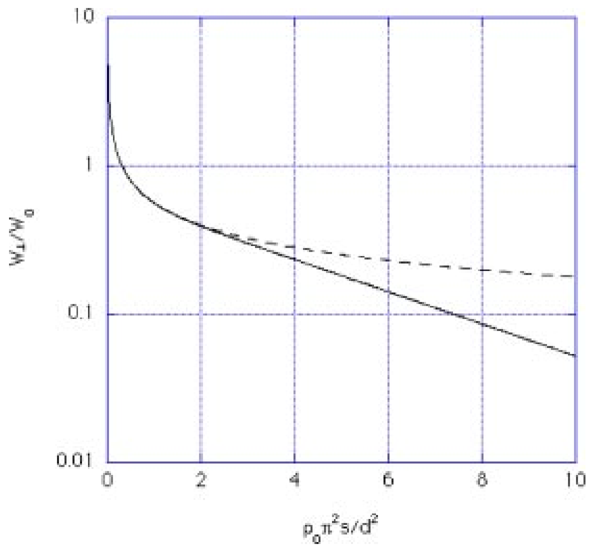

FIG. 2. The s dependence of the transverse wake function $W_\perp$ where $W_0 \equiv cZ_0\rho_0/(b^3 d)D_\perp$. —— corresponds to Eq. (21), and - - - to Eq. (22).

Here we take the inverse Fourier transformation of $F_\perp$ to obtain the transverse wake function $W_\perp$ per unit length,

$$W_\perp(s)=\int_{-\infty}^{\infty} dk e^{jks}\frac{c}{2\pi}\frac{Z_0}{2\pi b^3} \times D_\perp \frac{\delta(1+j)}{j}\frac{e^{\sqrt{2j}d/\delta}-e^{-\sqrt{2j}d/\delta}}{e^{\sqrt{2j}d/\delta}+e^{-\sqrt{2j}d/\delta}}. \quad (18)$$

The integrand in Eq. (18) does not have a cut, but simple poles at

$$k=k_n=j\frac{\rho_0\pi^2(2n+1)^2}{4d^2} \quad (n=0,1,2,\ldots). \quad (19)$$

By closing the contour by the upper semicircle for $s>0$,

$$W_\perp(s)=\frac{cZ_0}{2\pi^2 b^3}D_\perp j\pi\sum_{n=0}^{\infty} \quad \text{Residue } (k=k_n). \quad (20)$$

Thus, we obtain

$$W_\perp(s)=2\frac{cZ_0\rho_0}{\pi b^3}\frac{1}{d}\left[\sum_{n=0}^{\infty} e^{-(\pi^2\rho_0 s/d^2)(n+1/2)^2}\right]D_\perp. \quad (21)$$

Since the summation over n can be replaced by integration for the case $\rho_0 s/d^2 \ll 1$ (i.e., $\delta \ll d$), we can reproduce [2],

$$W_\perp(s)=\frac{cZ_0\sqrt{\rho_0}}{\pi\sqrt{\pi}b^3}\frac{1}{\sqrt{s}}D_\perp. \quad (22)$$

Comparing the asymptotic region of s in both Eqs. (21) and (22), we find that Eq. (21) deviates from $1/\sqrt{s}$ in the region $s \gtrsim 2d^2/(\pi^2\rho_0)$, and the wake function becomes exponentially damped (see Fig. 2). This is consistent with our other condition, $\delta \ll b$ (i.e., $s \lesssim b^2/2\rho_0$).

Here we divide $W_\perp(s)$ into the part proportional to the coordinate of the source particle (x_1 or y_1) and the part proportional to the coordinate of the witness particle (x or y), and write

$$W_\perp(s)=\begin{pmatrix} W_{1x}(s)x_1+W_2(s)x \\ W_{1y}(s)y_1-W_2(s)y \end{pmatrix}. \quad (23)$$

$W_{1x,y}(s)$ cause a coherent tune shift and $W_2(s)$ an incoherent tune shift. Since the equations of motion are

$$\frac{d^2x_n(s)}{ds^2}+K_x(s)x_n(s)=\sum_{m,k}\frac{Q}{E}[W_{1x}((n-m+kN)c\Delta t) \times x_m(s-kL)+W_2((n-m +kN)c\Delta t)x_n(s)], \quad (24)$$

$$\frac{d^2y_n(s)}{ds^2}+K_y(s)y_n(s)=\sum_{m,k}\frac{Q}{E}[W_{1y}((n-m+kN)c\Delta t) \times y_m(s-kL)-W_2((n-m +kN)c\Delta t)y_n(s)], \quad (25)$$

where $K_{x,y}$ are focusing forces, Q (C) is the charge per bunch, E is the electron (or, positron) energy (eV), $c\Delta t$ is the distance between bunches, L is the circumference of the ring and N is the total number of bunches. The coherent tune shift ($\nu^{(\mu)}_{coh,x,y}$) for μth mode and the incoherent tune shift ($\delta\nu_{inc,x,y}$) are given by

$$\delta\nu^{(\mu)}_{coh,x,y}=-\frac{L\langle\beta_{x,y}\rangle}{4\pi}\sum_{k=1}^{\infty}\frac{Q}{E}W_{1x,y}(kL)e^{j2\pi k\Delta\nu^{(\mu)}_{x,y}} \times\frac{\sin\pi\Delta\nu^{(\mu)}_{x,y}}{\sin\frac{\pi\Delta\nu^{(\mu)}_{x,y}}{N}}e^{j\pi\Delta\nu^{(\mu)}_{x,y}(1-1/N)}$$
$$=-\frac{L\langle\beta_{x,y}\rangle}{4\pi}\frac{Z_0 I_0}{E}\frac{c\Delta t\sqrt{\rho_0}}{\pi\sqrt{\pi}b^3}D_{1x,y}\sum_{k=1}^{\infty}\frac{2\sqrt{\pi\rho_0}}{d} \times\sum_{n=0}^{\infty} e^{-(\pi^2\rho_0 kL/d^2)(n+1/2)^2+j2\pi k\Delta\nu^{(\mu)}_{x,y}} \times\frac{\sin\pi\Delta\nu^{(\mu)}_{x,y}}{\sin\frac{\pi\Delta\nu^{(\mu)}_{x,y}}{N}}e^{j\pi\Delta\nu^{(\mu)}_{x,y}(1-1/N)}, \quad (26)$$

$$\delta\nu_{inc,x,y} = \mp \frac{L\langle\beta_{x,y}\rangle}{4\pi} \sum_{k=1}^{\infty} \frac{Q}{E} W_2(kc\Delta t)$$

$$= \mp \frac{L\langle\beta_{x,y}\rangle}{2\pi^2 b^3} \frac{Z_0 I_0}{E} \frac{c\Delta t \rho_0}{d}$$

$$\times D_{2xy} \sum_{n=0}^{\infty} \frac{1}{e^{(\pi^2\rho_0 c\Delta t/d^2)(n+1/2)^2} - 1}, \qquad (27)$$

where $\Delta\nu_{x,y}^{(\mu)} = \nu_{x,y} - \mu$, $\nu_{x,y}$ are tunes, I_0 (A) is the total current, $\langle\beta_{x,y}\rangle$ are average values of β functions around the ring, and the upper (lower) sign corresponds to $\delta\nu_{inc,x}$ ($\delta\nu_{inc,y}$).

Since $\pi\Delta\nu_{x,y}^{(\mu)} \ll 1$, $\pi^2\rho_0 L/d^2 \ll 1$, and $\pi^2\rho_0 c\Delta t/d^2 \ll 1$ in normal applications, Eqs. (26) and (27) become

$$\delta\nu_{coh,x,y}^{(\mu)} \simeq -\frac{L\langle\beta_{x,y}\rangle}{4\pi} \frac{Z_0 I_0}{E} \frac{L\sqrt{\rho_0}}{\pi\sqrt{\pi} b^3 \sqrt{2L}} D_{1x,y}$$

$$\times \sum_{k=1}^{\infty} \sqrt{\frac{2}{k}} \left\{ \cos\left[2\pi\Delta\nu_{x,y}^{(\mu)}\left(k+\frac{1}{2}\right)\right]\right.$$

$$\left. + j\sin\left[2\pi\Delta\nu_{x,y}^{(\mu)}\left(k+\frac{1}{2}\right)\right]\right\}, \qquad (28)$$

$$\delta\nu_{inc,x,y} \simeq \mp D_{2xy} \frac{L\langle\beta_{x,y}\rangle}{4\pi} \frac{Z_0 I_0}{E} \frac{1}{\pi b^3} d, \qquad (29)$$

where we have replaced the summation over n by an integral in Eq. (26) and used $\Sigma_{n=0}^{\infty} 1/(n+1/2)^2 = \pi^2/2$ in Eq. (27). Here we should notice that the coherent tune shift is automatically finite even in the limit $d\to\infty$ and does not depend on d even if we use Eq. (26). On the other hand, the incoherent tune shift is infinite for $d\to\infty$. Thus, we usually calculate the incoherent tune shift introducing a cutoff (N_Λ) in the number of bunches when we use Eq. (22). The tune shift is then written as

$$\delta\nu_{inc,x,y} = \mp D_{2xy} \frac{L\langle\beta_{x,y}\rangle}{4\pi} \frac{Z_0 I_0}{E} \frac{1}{\pi b^3} \sum_{j=1}^{N_\Lambda} \sqrt{\frac{\rho_0 c\Delta t}{\pi j}}. \qquad (30)$$

We can evaluate the effective cutoff. According to Eqs. (29) and (30), we obtain

$$c\Delta t N_\Lambda \simeq \frac{\pi d^2}{4\rho_0} \equiv c T_{max}. \qquad (31)$$

Equation (31) can be interpreted in terms of the skin depth, i.e., one should get k_{min} from

$$\sqrt{\frac{\pi}{2}} d = \delta(k_{min}), \qquad (32)$$

and define $cT_{max} \equiv 1/k_{min}$.

IV. APPLICATION

Let us compare experimental results at KEKB electron ring with our theory. In KEKB, parameters are given by $\nu_x = 44.514$, $\nu_y = 41.580$, $\langle\beta_{x,y}\rangle \simeq 11.0$ m, $L = 3016$ m, $E = 8\times 10^9$ eV, $d = 6\times 10^{-3}$ m, $a = 52\times 10^{-3}$ m, and $b = 25\times 10^{-3}$ m (a and b are defined in the Appendix). In KEKB, Ieiri measured the tune shift and got $d\nu_x/dI_0|_{meas} = 0.026$/A and $d\nu_y/dI_0|_{meas} = -0.037$/A [7]. Recently, a more precise measurement was done [8]. The tune-shift dependence was fitted by

$$\nu_x|_{meas} = -2.340\times I_b + 0.028\,06\times I_0 + 0.5195, \qquad (33)$$

$$\nu_y|_{meas} = -17.76\times I_b - 0.013\,83\times I_0 + 0.6045, \qquad (34)$$

where I_b (A) is the bunch current. Following Zimmermann [4], let us separate the tune shift into the coherent part and the incoherent part as follows:

$$\left.\frac{d\nu_x}{dI_0}\right|_{meas} = 0.5\frac{d\nu_{coh,r}}{dI_0} + \frac{d\nu_{inc,x}}{dI_0}, \qquad (35)$$

$$\left.\frac{d\nu_y}{dI_0}\right|_{meas} = 0.8\frac{d\nu_{coh,r}}{dI_0} + \frac{d\nu_{inc,y}}{dI_0}. \qquad (36)$$

Here $d\nu_{coh,r}/dI_0$ is the coherent tune shift for a round chamber. The numerical factors come from those for the resistive wake (Fig. 8 of Ref. [2] for $(a-b)/(a+b)\simeq 0.35$). We expect a similar ratio with some uncertainties even if the relevant wake is not of resistive wall type. From the data we obtain

$$\frac{d\nu_{coh,r}}{dI_0} \simeq 0.010\,95/\text{A}, \quad \frac{d\nu_{inc,x,y}}{dI_0} \simeq \pm 0.022\,59/\text{A}. \qquad (37)$$

Using our theory, $T_{max} \simeq 1.9$ ms. According to Eqs. (28) and (29), the tune shift is given by

$$\frac{d\nu_{coh,x}}{dI_0} \simeq 0.000\,016/\text{A}, \quad \frac{d\nu_{coh,y}}{dI_0} \simeq 0.000\,148/\text{A},$$

$$\frac{d\nu_{inc,x,y}}{dI_0} \simeq \pm 0.0060/\text{A}. \qquad (38)$$

We cannot explain the coherent tune shift by a resistive wake. We should consider the other sources of impedances. For the incoherent tune shift, our value is about four times smaller than the data [Eq. (37)]. This discrepancy is significant even if we take into account the uncertainty of the coherent part. In our case, the effect of the radiation damping is negligible, because the damping time (46 ms) is much larger than T_{max}. The reason of the discrepancy may be (1) other sources of impedances, (2) the effects of materials outside the chamber. Since the observation showed that the tune shift is proportional to the total current rather than to the bunch current, the responsible wake has to be a long range wake, but does not seem to be of narrow resonance type. Further, the nonround cross section must be related to this wake, because the sign of the tune shift is opposite for x and y. We think that case (1) is impossible, therefore, we should consider the possibility of (2), because the fields leak out of the chamber on a large time scale. The next step will be to consider the effects of these components outside the chamber.

V. SUMMARY

Resistive wall impedances with a finite thickness can be obtained by multiplying the $\mathcal{G}$ factor by impedances with infinite thickness under the approximation that the size of the chamber is sufficiently larger than a skin depth. This transverse wake function becomes exponentially damped for $s > 2d^2/\pi^2\rho_0$ rather than falling like $1/\sqrt{s}$. The expression for an incoherent tune shift is automatically finite by using this impedance. Thus, there is no ambiguity to evaluate the tune shift. The reason why the divergence of the tune shift is resolved, is that the physically important parameter is the thickness of the beam pipe instead of the skin depth for a long time scale. By reducing the thickness of the chamber, we can reduce the tune shift linearly.

In this paper, we also try to explain the incoherent tune shift which is observed in KEKB. Our value is four times smaller than the experimental data. This situation suggests that we have to consider the effect of components that exist outside the chamber: for example, magnets. The next step will be to consider the effects of these components to the tune shift.

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APPENDIX: *D* FUNCTIONS

Here, we denote the size of a cross section of a chamber as follows. The upper value holds for the case where the cross section is elliptic, its major axis is $2a$ and minor axis is $2b$; the lower value holds for the case where the cross section is rectangular, its boundaries are given by $x = \pm a$ and $y = \pm b$:

$$D_0 = \begin{cases} G_0(u_0) = \dfrac{\sinh u_0}{2\pi}\displaystyle\int_0^{2\pi} dv\, \frac{Q_0^2(v)}{\sqrt{\sinh^2 u_0 + \sin^2 v}} & \text{for elliptic case} \\ F_0\left(\dfrac{b}{a}\right) = \pi \displaystyle\sum_{m=1,odd} \left(\frac{1}{\cosh^2 \frac{m\pi a}{2b}} + \frac{b}{a}\, \frac{1}{\cosh^2 \frac{m\pi b}{2a}} \right) & \text{for rectangular case,} \end{cases}$$

$$D_{1x} = \begin{cases} G_{1x}(u_0) = \dfrac{\sinh^3 u_0}{4\pi}\displaystyle\int_0^{2\pi} dv \frac{Q_{1x}^2(v)}{\sqrt{\sinh^2 u_0 + \sin^2 v}} & \text{for elliptic case,} \\ F_{1x}\left(\dfrac{b}{a}\right) = \dfrac{\pi^3}{8} \left(\displaystyle\sum_{m=1,odd} \frac{m^2}{\sinh^2 \frac{m\pi a}{2b}} + \sum_{m=2,even} \frac{b^3 m^2}{a^3 \cosh^2 \frac{m\pi b}{2a}} \right) & \text{for rectangular case,} \end{cases}$$

$$D_{1y} = \begin{cases} G_{1y}(u_0) = \dfrac{\sinh^3 u_0}{4\pi}\displaystyle\int_0^{2\pi} dv \frac{Q_{1y}^2(v)}{\sqrt{\sinh^2 u_0 + \sin^2 v}} & \text{for elliptic case} \\ F_{1y}\left(\dfrac{b}{a}\right) = \dfrac{\pi^3}{8} \left(\displaystyle\sum_{m=1,odd} \frac{b^3 m^2}{a^3 \sinh^2 \frac{m\pi a}{2b}} + \sum_{m=2,even} \frac{m^2}{\cosh^2 \frac{m\pi b}{2a}} \right) & \text{for rectangular case,} \end{cases}$$

$$D_{2xy} = \begin{cases} \dfrac{\sinh^3 u_0}{4\pi}\displaystyle\int_0^{2\pi} dv \frac{Q_0(v) Q_{2xy}(v)}{\sqrt{\sinh^2 u_0 + \sin^2 v}} & \text{for elliptic case} \\ \dfrac{\pi^3}{8} \displaystyle\sum_{m=1,odd} \left(\frac{m^2}{\cosh^2 \frac{m\pi a}{2b}} - \frac{b^3 m^2}{a^3 \cosh^2 \frac{m\pi b}{2a}} \right) & \text{for rectangular case,} \end{cases} \tag{A1}$$

where

$$Q_0(v)=1+2\sum_{m=1}^{\infty}(-1)^m\frac{\cos 2mv}{\cosh 2mu_0},\quad Q_{1x}(v)=2\sum_{m=1}^{\infty}(-1)^m(2m+1)\frac{\cos(2m+1)v}{\cosh(2m+1)u_0},$$

$$Q_{1y}(v)=2\sum_{m=1}^{\infty}(-1)^m(2m+1)\frac{\sin(2m+1)v}{\sinh(2m+1)u_0},\quad Q_{2xy}(v)=-8\sum_{m=1}^{\infty}(-1)^m\frac{m^2\cos 2mv}{\cosh 2mu_0},$$

$$a=f\cosh u_0,\quad b=f\sinh u_0\quad \text{for elliptic case.} \tag{A2}$$

Functions $G_{0,1x,1y}(u_0)$, $F_{0,1x,1y}(\lambda)$, and $Q_{0,1x,1y}(v)$ are calculated by GZZ [1].

[1] R. L. Gluckstern, J. van Zeijts, and B. Zotter, Phys. Rev. E **47**, 656 (1993).
[2] K. Yokoya, Part. Accel. **41**, 221 (1993).
[3] S. Heifets, Report No. SLAC-PUB 7985, 1998 (unpublished).
[4] F. Zimmermann, Commissioning report, 2002 (unpublished).
[5] See, e.g., J. K. Jackson, *Classical Electrodynamics* (Wiley, New York, 1974), p. 432.
[6] A. W. Chao, *Physics of Collective Beam Instabilities in High Energy Accelerators* (Wiley, New York, 1993).
[7] T. Ieiri *et al.*, in *Proceeding of the Second Asian Particle Accelerator Conference, Beijing*, edited by S. Fang *et al.* (IHEP, Beijing, 2001). p. 412.
[8] T. Ieiri (private communication).